load is applied to their face. When the diameter of the disk is sufficiently great in comparison to its thickness, and a suitable guard-ring electrode configuration is used, the fields throughout the inner region of the sample will be normal to the faces of the disk [65G1].

In the absence of free charge in the disk, the electric displacement will be independent of position, although it will vary with time: $\boldsymbol{D} = (D(t), 0, 0)$. The current induced in the external circuit is attributable to changes in electric displacement within the disk and is given by

$$i(t) = A\,\mathrm{d}D(t)/\mathrm{d}t, \tag{4.3}$$

where A is the area of the charge-collecting electrode on the face of the disk. The voltage across the disk is the integral of the electric field in the space between the electrodes,

$$V(t) = \int_{\hat{x}(0,t)}^{\hat{x}(L,t)} E(x,t)\,\mathrm{d}x. \tag{4.4}$$

When the electrodes are connected by a short circuit, the voltage is zero but only the average field need vanish and local values are often quite large.

Piezoelectric response to shock compression: uncoupled short-circuit solution. The "uncoupled" response of a piezoelectric sample to elastic shock compression is determined on the assumption that the mechanical response of the material is independent of any electric fields that may be present. In this approximation, a steady shock introduced into material at rest in its reference configuration will advance at some constant velocity U and divide the thickness of the sample into two parts. The part ahead of the shock will remain undeformed and at rest while the part behind the shock will be uniformly compressed to a strain $S_1 = -u/U$, where u is the particle velocity of the compressed material. The compression will produce a piezoelectric polarization $\boldsymbol{P}^{\eta} = (P^{\eta}, 0, 0)$ in the compressed material and will cause a change in the permittivity component in the x direction from some value ε_{11} to a new value $\bar{\varepsilon}_{11}$ that depends on the strain according to eq. (4.2). From equations (4.2)–(4.4), it can be shown that, when the electrodes are connected by a short circuit ($V(t) = 0$) [72G3],

$$\frac{i(t)L}{P^{\eta}AU} = \frac{\alpha(1 - u/U)}{[(1 - u/U)(t/t_0) + \alpha(1 - t/t_0)]^2}, \quad 0 < t < t_0 \tag{4.5}$$

where L is the original thickness of the sample, $t = 0$ at the instant of introduction of the shock into the sample, $t_0 = L/U$ is the transit time of the shock through the sample, and $\alpha = \bar{\varepsilon}_{11}/\varepsilon_{11}$. From eq. (4.2), we see that the magnitude of the piezoelectric polarization is directly dependent on the magnitude of the strain but eq. (4.5) shows that this dependency does not affect the current history for the case considered. For both quartz and lithium niobate, $u/U < 0.04$ and $1 \le \alpha < 1.01$ in the elastic range. With these restrictions, eq. (4.5) indicates that the current history is a step function to close approximation. Deviations from this ideal form occur as a geometrical result of large compressions or when the permittivity changes significantly upon compression.

The current immediately after impact, $i(0+)$, is calculated from the limiting case

$$\frac{i(0+)L}{AU} = \frac{1 - (u/U)}{\alpha} P^{\eta} \tag{4.6}$$

of eq. (4.5). This relation holds universally for shock-induced polarization effects and is useful for evaluation of various shock-induced polarization phenomena.

In the low-signal limit in which nonlinearities in material behavior are negligible and $u/U \ll 1$ the analysis given above can easily be extended to stress pulses of arbitrary form, with the result [65G1]

$$\frac{i(t)L}{AU} = \frac{e_{111}}{C^E_{1111}} t_{11}(0,t), \qquad 0 < t < t_0, \tag{4.7}$$

which indicates that the current history is proportional to the history of stress at the input electrode, $t_{11}(0,t)$. This relation, which is also followed to a close approximation at larger strains if e_{111}/C^E_{1111} is replaced by an experimentally-determined strain-dependent coefficient, forms the basis for the widely used current-mode piezoelectric gauges [65G1, 75G4].

Electric fields. Shock compression of piezoelectric solids, even under short-circuit conditions, causes large electric fields of varying amplitude and polarity within the material. In the uncoupled approximation to the solution of the short-circuit problem, the field is easily determined from the condition on uniformity of electric displacement, from eq. (4.4), and the expression $\boldsymbol{D} = \boldsymbol{P}^{\pi} + \varepsilon \boldsymbol{E}$ [74G2]

$$E = \frac{P^{\pi}/\varepsilon_{11}}{(1 - u/U)(t/t_0) + \alpha(1 - t/t_0)} \begin{Bmatrix} (1 - u/U)(t/t_0), & \text{for } x > Ut \\ 1 - t/t_0, & \text{for } x < Ut \end{Bmatrix} \tag{4.8}$$

for $0 < t < t_0$. The magnitude of the field in each region varies between zero and a maximum value of P^{π}/ε_{11} during passage of the shock through the sample. Since u/U is small and α near unity for most cases of interest, the field at a given point varies approximately linearly with time except for a discontinuity when the shock passes. When a 2 GPa shock passes through X-cut quartz, the maximum field strength is about 10^8 V/m, while a maximum field of 3×10^7 V/m is realized in Z-cut lithium niobate subjected to a stress slightly in excess of 1 GPa. Fields of these magnitudes are of concern in that they are about one-tenth of the breakdown strength values at atmospheric pressure. From this analysis we see that shock-compressed piezoelectric materials are subjected to the simultaneous effects of high stress and high electric field. As discussed in section 4.6, such fields are crucial in activating shock-induced conduction.

The electric fields for stress pulses of arbitrary profile can be evaluated by noting that eq. (4.7) gives an expression relating a measured current history to the piezoelectric polarization $\boldsymbol{P}^{\pi}(0,t)$ at the input electrode. Combined with eq. (4.3) and the relation, $\boldsymbol{D} = \boldsymbol{P}^{\pi} + \varepsilon \boldsymbol{E}$, the measured current and its integral can then be used to calculate the fields in a point-by-point approximate solution. *Piezoelectric pulse diagrams* in which expressions for polarization and displacement versus time are superimposed and the field determined as their differences, provide graphical solutions for the time dependence of the fields. Consideration of the simple case of a square pulse whose duration is less than t_0 shows that the field can be varied independently of the stress (see section 4.6).

Weakly- and fully-coupled solutions. Uncoupled solutions for current and electric field give simple and explicit descriptions of the response of piezoelectric solids to shock compression, but the neglect of the influence of the electric field on mechanical behavior (i.e., the electromechanical coupling effects) is a troublesome inconsistency. A first step toward an improved solution is a weak-coupling approximation in which it is recognized that the effects of coupling may be relatively small in certain materials and it is assumed that electromechanical effects can be treated as a perturbation on the uncoupled solution.